A quantification method for shale fracability based on analytic hierarchy process

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ABSTRACT
A quantification method for evaluation of the fracability of shale is essential for optimizing hydraulic fracturing of shale gas reservoirs and enhancing shale gas recovery. To quantitatively evaluate the fracability, seven sets of shale cores are drilled from the reservoirs at different depths of an oilfield in the east of China. The influences of six fracability-related mechanical and physical characteristics of shale, i.e. brittleness, brittle mineral content, clay mineral content, cohesion, angle of internal friction, and unconfined compressive strength, are analyzed. A mathematical model taking account of significance of the influencing factors is proposed based on analytic hierarchy process (AHP) to evaluate the fracability according to their different effects on shale fracability. The analysis indicates that the fracability decreases with the increase of reservoir depths. The hydraulic fracturing tests of the shale cores are conducted to verify the accuracy of the quantification method. The fractal geometry is used to characterize the fracture degree of the shale. It is shown that a larger fractal dimension of the fracture network corresponds to a better fracability of shale. The more complex the fractures are, the larger the fracability of shale is. The experimental data coincide with the results of the proposed evaluation model.

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1. Introduction

Hydraulic fracturing, as one of the most popular means for extracting unconventional shale gas, has been adopted to effectively stimulate shale gas reservoirs. The purpose of hydraulic fracturing is to fracture shale reservoirs to create high-conductivity paths for gas migration, and thus enhancing gas productivity [1–3]. Fracability is an index that has been used to evaluate whether a shale reservoir is easy to be fractured and form developed fracture networks [4]. A higher fracability means that the shale reservoir may be more easily to be fractured and have a greater gas production rate. The fracability has become a theoretical measure to evaluate the exploitation potential of shale gas reservoirs.

Originally, most researchers adopted formation brittleness alone to evaluate the fracability of unconventional shale reservoirs [5–8]. It was assumed that a formation with high brittleness is easy to be fractured into developed fracture networks. Various brittleness evaluation methods have been proposed accordingly. For instance, Sondergeld et al. first proposed a brittleness index formula using the content of quartz [9]. Wang and Gale took dolomite as an additional brittle mineral and defined the material brittleness using the percentage of two mineral components [10]. Altamar defined the reservoir brittleness using the contents of feldspar, dolomite and quartz [11]. As research continues, more and more researchers realize that the fracability is affected not only by material brittleness but also by other factors. Gökten pointed out that, in addition to brittleness, the specific energy should be added into fracability evaluation method [12]. Breyer et al. [13] and Bruner et al. [14] believed that fracability should be a function of material brittleness and ductility, since a formation with higher brittleness may have higher energy barrier in the same time, which results in a dilemma that the reservoir has a high brittleness but a low fracability. Jin et al. integrated brittleness and energy dissipation of the reservoir in their fracability evaluation model [15]. Ding et al.
divided the influencing factors of fracability into the structural factors and the non-structural factors so that the different influences of factors can be evaluated [16]. Guo et al. defined fracabil-
yity using the angle of internal friction, fracture toughness and brittleness of the reservoir [17], and regarded the high angle of internal friction as an adverse factor for fracability. On the contrary, quite a few researchers believe that a larger angle of internal friction results in a higher fracability [18–20]. Tang et al. adopted four parameters of a rock reservoir, i.e. brittleness, quartz content, natural fractures, and diagenesis, to quantitatively describe the frac-
ability [21]. Yuan et al. adopted the elastic modulus, Poisson’s ratio, and tensile strength of the rock formation as independent variables to evaluate the fracability [22]. Guo et al. utilized the fractal dimension of the fracture network of a reservoir to illustrate that rock brittleness, hardness and formation deposition can be used to evaluate the fracability, but no quantitative formulation was derived to link the parameters [23]. Wang et al. proposed a comprehensive fracability model considering the influences of brittleness, natural fracture, stress sensitivity and acoustic emission, but these four factors were treated as equally influencing factors [24].

Despite the development in quantification methods for rock fracability, the existing methods are controversial, mainly because there are no sufficient shale cores for quantification tests due to the technical difficulties and high costs in extracting shale cores from the fields [25,26]. Moreover, researchers have realized that there are some barriers that need to be overcome when developing an effective quantification method for shale fracability. First, selecting the parameters to evaluate rock fracability is random, and lacks theoretical basis and widespread applicability for various rock formations. Second, establishing a quantification model is highly dependent on acquisition of the multiple influencing factors through complicated laboratory tests. This brings about inconvenience to practical applications of models. Thus, for the sake of easy application, a simple, feasible quantification model taking into account the effects of influencing factors is needed. This study aims to establish an analytical model for quantitatively evaluating rock fracability by means of analytic hierarchy process (AHP) which takes into account the effects of fracability-related factors of shale rock. The simplicity and practicability of the quantification model is emphasized.

The remainder of this paper is structured as follows. In Section 2, the effects of the potential primary influencing factors on shale fracability are first analyzed based on existing studies. Then, in Section 3, a series of laboratory tests on a few shale cores extracted from an oilfield in the east of China are conducted to measure the influencing factors of fracability. In Section 4, the analytic hierarchy process (AHP) is introduced and applied to establish a comprehensive model for evaluating the fracability of the shale rock according to the significance of the parameters on fracability. In Section 5, a triaxial hydraulic fracturing test is performed on a couple of shale cores to create the fracture network to which the CT imaging technique and fractal geometry is applied for characterizing the complexity of fractures. The feasibility and accuracy of the proposed evaluation method is verified through comparing the evaluation results with the fractal characterization of the fracture network. The concluding remarks are summarized in Section 6.

2. Analysis of the influencing factors of rock fracability

In the existing quantification models for rock fracability, the brittleness, mineral content, cohesion, angle of internal friction and unconfined compressive strength were usually taken as the primary factors that significantly influence the fracability.

2.1. Brittleness

As aforementioned, brittleness is the first selected factor to characterize the fracability of rock. Brittleness has been confused with fracability for a long time [5–11,27,28]. There has been no universally spread definition to quantitatively measure rock brittleness. For instance, Morley and Hetenyi defined the brittleness as the lack of material plasticity [29,30]. Ramsey considered that rock brittle break happened once rock cohesion was destroyed [31]. Different definitions result in a variety of quantification methods for rock brittleness [6,24,32,33]. Table 1 lists some popular methods.

Despite of the inconsistency in calculation of rock brittleness, the wide-spread belief is that the higher the brittleness of rock is, the easier the rock can be fractured. Through comparing the different methods, we noticed that the method adopting the Young’s modulus and Poisson’s ratio of rock is more straightforward and simpler to define rock brittleness than any other methods. This has been verified by Rickman’s work [35]. In the Rickman’s model, the brittleness of rock is defined as:

\[ BI = (YM_Br + PR_Br)/2 \]  
\[ YM_Br = (YM_r - 1)/(8 - 1) \]  
\[ PR_Br = (PR_r - 0.4)/(0.1 - 0.4) \]

where \( BI \) refers to the rock brittleness, \( YM_r \) is the Young’s modulus with unit of 10 GPa, \( PR_r \) is the Poisson’s ratio, \( YM_Br \) is the normalized Young’s modulus, and \( PR_Br \) is the normalized Poisson’s ratio. From the definition, it is straightforward to understand that a high Young’s modulus and a low Poisson’s ratio can lead to a high brittleness. Therefore, for the purpose of characterizing rock fracability, Young’s modulus and Poisson’s ratio are used to determine the brittleness of rock in this study.

2.2. Brittle mineral content and clay mineral content

Brittle mineral content and clay content are two important factors affecting the porosity, development of micro fractures, hydrocarbon content and fracturing ways of shale [36]. Higher content of brittle materials makes rock formations more brittle and thus being easily fractured [37]. On the contrary, higher content of clay minerals makes rock formation more plastic and absorb more energy. High clay content is believed to be a barrier to fracturing shale formations [38]. Nelson’s and Tanaka’s studies imply that the brittle minerals comprise quartz, feldspar and dolomite [39], and the clay minerals family contains illite, montmorillonite, kaolinite, pyrophyllite and talc [40]. Besides brittle and clay minerals, there are some neutral materials that have no effect on rock fracability [36–40]. According to the experimental results of existing studies, we have noticed that the categories and contents of minerals vary with geographical locations of shale reservoirs. Considering the geographical features of the shale formation in our study, the illite and smectite mixed minerals and illite are identified experimentally.

2.3. Cohesion

In general, cohesion refers to the component of shear strength of a rock when no normal stress is applied, and reflects the ability of the rock against shear sliding between adjacent sections. The Mohr-Coulomb’s criterion indicates that fracture occurs only when the maximum shear stress reaches a level that surpasses the rock
2.4. Angel of internal friction

Regarded as a negative index of rock fracability, the angel of internal friction indicates the difficulty level of rock sliding along the failure surface. The smaller the angle of internal friction is, the easier the rock slides along the failure surface. Before rock breaks, the ability against sliding along the failure surface of brittle or hard rock is greater than plastic or soft rock. Thus, the angle of internal friction is a positive factor which quantifies the level of rock fracability [18–20].

2.5. Unconfined compressive strength

It is shown that mineral composition apparently influences the compressive strength of reservoir rock. Quartz is one of the known minerals that has high intensity. The rock containing a large amount of quartz exhibits high brittleness. Breyer combined the unconfined compressive strength and the angle of internal friction to further refine rock brittleness [13]. Huacka and Das [18] proposed the ratio of the unconfined compressive strength and the tensile strength to define rock brittleness. They stated that the brittle rock has a high ratio of the unconfined compressive strength and the tensile strength. Considering that brittle rock generally has a low tensile strength [42], the unconfined compressive strength is predominately related to brittleness and fracability of rock. The higher the unconfined compressive strength is, the easier the rock is fractured. The unconfined compressive strength of rock can be taken as a positive indicator of fracability.

2.6. Contribution of each factor to rock fracability

According to the above analysis of the effects of the six influencing factors on fracability, the positive and negative influencing results can be summarized as shown in Table 2. It indicates that the listed six factors are closely related to the fracability of reservoir rock. Thus, they should be included in the quantification model of rock fracability.

It should be noted that the geological effects of rock formations such as diagenesis and deposition are not included in the above factorial analysis since we consider that the shale cores for the experiments and quantification analysis of fracability in this study are extracted from the same exploration area. There is no significant difference between the geological depths of the shale reservoirs where the core samples are drilled out (Details refer to the following tests). The diagenesis and depositional environment of the tested core samples is similar, so we didn’t take into account the effects of geological factors in our analysis and following quantification model. Nonetheless, it is feasible to add them in our mathematical model to consider their potential effects on the fracability of rocks from different formations (See the following model and analysis).

3. Experimental measurements of the influencing factors

3.1. Preparation of shale core specimens

To acquire the properties of the influencing factors, seven sets of shale cores were drilled from different formations with depths within 3317–3485.07 m at an oilfield in east China. The original cores were the cylinders with 108–110 mm in diameters and 150 mm in length. 7–10 small cylindrical specimens with 25 mm in diameter and 50 mm in length were finished from each core set for the tests to measure the properties of the influencing factors. The remaining original cores were used for hydraulic fracturing tests to examine the fracability and the accuracy of the quantification model. The shale cores were categorized according to their depths, such as diagenesis and deposition are not included in the above factorial analysis since we consider that the shale cores for the experiments and quantification analysis of fracability in this study are extracted from the same exploration area. There is no significant difference between the geological depths of the shale reservoirs where the core samples are drilled out (Details refer to the following tests). The diagenesis and depositional environment of the tested core samples is similar, so we didn’t take into account the effects of geological factors in our analysis and following quantification model. Nonetheless, it is feasible to add them in our mathematical model to consider their potential effects on the fracability of rocks from different formations (See the following model and analysis).

### Table 1
Calculation methods for rock brittleness.

<table>
<thead>
<tr>
<th>Calculation methods</th>
<th>Variable definitions</th>
<th>Test methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_1 = (\sigma_v/\sigma_t)[32] )</td>
<td>( \sigma_v ): Uniaxial compressive strength, ( \sigma_t ): Uniaxial tensile strength</td>
<td>Uniaxial compression and Brazilian tests</td>
</tr>
<tr>
<td>( B_2 = P_{ave}/P_{ave}[33] )</td>
<td>( P_{ave} ): Average force increment, ( P_{ave} ): Average force decrement</td>
<td>Indentation tests</td>
</tr>
<tr>
<td>( B_3 = W_{dol}/W_{tot}[6] )</td>
<td>( W_{dol} ): Weight of dolomite, ( W_{tot} ): Total mineral weight</td>
<td>Mineralogical logging or XRD tests</td>
</tr>
<tr>
<td>( B_4 = (W_{dol} + W_{quart})/W_{tot}[24] )</td>
<td>( W_{dol} ): Weight of dolomite, ( W_{quart} ): Weight of quartz</td>
<td>Mohr circles or logging data</td>
</tr>
<tr>
<td>( B_5 = s_{int}[18] )</td>
<td>( s_{int} ): Angle of internal friction</td>
<td>Uniaxial compression tests</td>
</tr>
<tr>
<td>( B_6 = (E_n + v_n)[34,35] )</td>
<td>( E_n ): Normalized dynamic Young’s modulus, ( v_n ): Normalized dynamic Poisson’s ratio</td>
<td></td>
</tr>
</tbody>
</table>

Note that the symbol \( B_i \) refers to rock brittleness.

### Table 2
Contribution results of the influencing factors.

<table>
<thead>
<tr>
<th>Effects</th>
<th>Britteness</th>
<th>Brittle mineral content</th>
<th>Clay mineral content</th>
<th>Cohesion</th>
<th>Angel of internal friction</th>
<th>Unconfined compressive strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Negative</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
and identified to Shale #1 to Shale #7, as shown in Table 3. It is noted that the core specimens labeled Shale #6 and Shale #7 were extracted from the formations with the same depths and designed for verifying the accuracy of the fracability model since their properties are close.

3.2. Experiments for determining the properties of the influencing factors

Fig. 1 presents the high-rigidity digital servo-control testing unit for measuring the uniaxial properties including Young’s modulus, Poisson’s ratio and unconfined compressive strength, the X-ray diffraction device for analyzing the mineral composition, and the triaxial testing system for acquiring the cohesion and the angle of internal friction of the shale.

To accurately measure the Young’s modulus, Poisson’s ratio and unconfined compressive strength of the shale, the displacement loading mode with a rate 0.001 mm/s was applied to the uniaxial compression test. The unconfined stress-stain curves and the compressive strengths were recorded using the device. Three cylindrical specimens from each core set were repeatedly tested in order to attain the proper representative values of the desired properties of shale. The brittleness of shale can then be evaluated using Eq. (1) once the desired Young’s modulus, Poisson’s ratio and unconfined compressive strength were obtained through this experiment.

The X-ray diffraction experiment was implemented to determine the contents of the brittle minerals including quartz, feldspar and dolomite, and the clay minerals including illite and smectite mixed mineral of shale. The triaxial compression test was conducted to measure the cohesion and the angle of internal friction of shale. The details of experimental setup can refer to the literature [43,44]. The cohesion and the angle of internal friction can be obtained using the relationship curve of $(\sigma_1-\sigma_3)$ and the following Eqs. (4) and (5)

\[
\varphi = \sin^{-1}\left(\frac{(m-1)}{(m+1)}\right) \\
c = -b(1 - \sin\varphi)/(2\sin\varphi)
\]

where $\varphi$ means the angle of internal friction ($^\circ$), $c$ means the cohesion (MPa), $m$ is the slope and $b$ is the intercepts of the relationship curve [45].

3.3. Experimental results

Table 3 lists the experimental results of Young’s modulus and Poisson ratio of shale cores. The brittleness indices, as determined using Eq. (1), for various shale cores with different buried depths are also included. Table 4 outlines the measured properties of the six factors.

4. Quantification methods and models

4.1. Principles of analytic hierarchy process (AHP)

The method of analytic hierarchy process (AHP) was initiated by Saaty [46] in the mid-1970s, which has been widely used for solving choice and ranking problems. One of the most favorite features of AHP is the easy and efficient determination of the relative weights of multiple factors for a comprehensive evaluation problem. Thus, we adopt this method to quantitatively evaluate the fracability of shale according to the measured results.

In general, application of AHP consists of four steps [47]:

① Constructing a hierarchy that represents the decisional problem

This step is to select the most influential factors on the problem to be solved and construct them in a hierarchy with three levels: goal, criteria and attributes.

② Evaluating a pairwise comparison matrix (subjective analysis according to experiences)

Considering a matrix $A=(a_{ij})$ that shows the results of the pairwise comparison of the factors according to the subjective experience and judgement of experts, as shown in Table 5, where the number $a_{ij}$ scales the contribution of the factor $i$ over the factor $j$ to the objective.

③ Deriving the weight of each factor

Fig. 1. Photographs of the testing devices for measuring the uniaxial compressive properties (a), the mineral composition (b), and the triaxial compressive properties (c) of shale.
The weight value of each factor could be determined using the sorting equation of the asymptotic normalization coefficient (ANC). The weight value $w_i$ can be calculated using

$$w_i = \frac{\sum_{j=1}^{n} a_{ij}}{n} \quad (i = 1, 2, 3, \cdots, n),$$

(6)

where the number $a_{ij}$ scales the significance of factors in the matrix.

© Evaluating and ranking the candidates

This step is designed to identify the most suitable candidates to the problem to be solved or rank all the candidates according to the value of $F(i) = \sum_{j=1}^{6} w_{ij} x_{ij} \quad (i = 1, 2, 3, \cdots, n)$, where $x_{ij}$ is the value of the $j$th influencing factor of the $i$th candidate, and $F(i)$ means the evaluation result of the objective contributed by the $i$th candidate. In this study, the objective $F(i)$ refers to the rock fracability.

4.2. Significance of the influencing factors

Considering that the above influencing factors are characterized by different dimensions, magnitudes and effective ranges and in order to use the weighted average values to evaluate the fracability using AHP method, the following equation is used to normalize the six factors. The positive and negative effects of factors can be determined using Table 2.

$$x_j = \begin{cases} \frac{(X - \text{min}X)}{(\text{max}X - \text{min}X)} & \text{for positive factors} \\ \frac{(\text{max}X - X)}{(\text{max}X - \text{min}X)} & \text{for negative factors} \end{cases}$$

(7)

where $X$ means the normalized factor, $\text{max}X$ and $\text{min}X$ are the maximum and the minimum values of the influencing factors. The normalized factor $x_j \quad (j = 1, 2, 3, \cdots, 6)$ represents the brittleness, brittle mineral content, cohesion, clay mineral content, angle of internal friction and unconfined compressive strength of shale, respectively. Table 6 lists the normalized influencing factors of various shale reservoirs, in which the factor of brittleness is not normalized because it ranges from 0 to 1, and it is dimensionless too. According to the normalized values and the analysis of influence of the factors in Section 2, it can be obtained that the normalized value of the factor close to 1.0 indicates that the factor has significant influence on the fracability. If the normalized value is close to 0, it means the factor has little influence on the fracability.

4.3. Evaluation of fracability

AHP is a decision-making method taking use of the subjective assessment to determine the influencing factors’ weights [47,48]. To apply AHP method, the significance of the six factors as for fracability is first compared and scaled, i.e. the significance scale $a_{ij}$ of one factor with respect to another is determined. A matrix $(a_{ij})$ of the influencing factor’s scales, shown in Table 7, is then constructed to determine the weight of each factor using Eq. (6). According to the parametric evaluation through the majority of the existing studies and the above analysis of influencing factors of fracability in Section 2, in the scale matrix, brittleness is treated as the most significant influencing factor of the fracability of shale rock, successively followed by brittle mineral contents, cohesion and clay mineral contents, considering that brittleness and brittle mineral contents directly and significantly influence rock fracability and cohesion and clay mineral contents take their effects on fracability based on their influences on the brittle properties. The indices of

Table 4

<table>
<thead>
<tr>
<th>Shale #1</th>
<th>Shale #2</th>
<th>Shale #3</th>
<th>Shale #4</th>
<th>Shale #5</th>
<th>Shale #6</th>
<th>Shale #7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Britteness</td>
<td>Brittle mineral content (%)</td>
<td>Clay mineral content (%)</td>
<td>Cohesion (MPa)</td>
<td>Angle of internal friction (°)</td>
<td>Unconfined compressive strength (MPa)</td>
<td></td>
</tr>
<tr>
<td>0.49</td>
<td>72.6</td>
<td>14.8</td>
<td>25.0</td>
<td>17.0</td>
<td>99.15</td>
<td></td>
</tr>
<tr>
<td>0.38</td>
<td>26.7</td>
<td>27.2</td>
<td>25.5</td>
<td>21.8</td>
<td>46.64</td>
<td></td>
</tr>
<tr>
<td>0.29</td>
<td>61.0</td>
<td>13.7</td>
<td>14.8</td>
<td>19.6</td>
<td>66.19</td>
<td></td>
</tr>
<tr>
<td>0.43</td>
<td>25.3</td>
<td>15.9</td>
<td>11.9</td>
<td>21.7</td>
<td>39.18</td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>28.9</td>
<td>21.7</td>
<td>14.6</td>
<td>22.9</td>
<td>91.66</td>
<td></td>
</tr>
<tr>
<td>0.29</td>
<td>10.7</td>
<td>18.6</td>
<td>13.2</td>
<td>20.6</td>
<td>78.10</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>14.3</td>
<td>11.1</td>
<td>34.3</td>
<td>23.6</td>
<td>59.80</td>
<td></td>
</tr>
</tbody>
</table>

Note that the calculated values are the averages of the tested results of 2–3 shale samples.

Table 5

<table>
<thead>
<tr>
<th>Numerical scale</th>
<th>Meaning and explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{ij}$</td>
<td>Factors i and j contribute equally to the objective</td>
</tr>
<tr>
<td>1</td>
<td>Factor i is favored slightly over factor j</td>
</tr>
<tr>
<td>3</td>
<td>Factor i is favor very strongly over factor j</td>
</tr>
<tr>
<td>5</td>
<td>Factor i is extremely important over factor j</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>The intermediate values reflect intermediate position of importance</td>
</tr>
<tr>
<td>Reciprocals (i.e. 1/3, 1/5, 1/7, 1/9)</td>
<td>The reciprocal number reflects the reverse comparison positions of above</td>
</tr>
</tbody>
</table>

Table 6

<table>
<thead>
<tr>
<th>Shale category (i)</th>
<th>Normalized factors ($x_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_1$</td>
</tr>
<tr>
<td>Shale #1</td>
<td>0.49</td>
</tr>
<tr>
<td>Shale #2</td>
<td>0.38</td>
</tr>
<tr>
<td>Shale #3</td>
<td>0.29</td>
</tr>
<tr>
<td>Shale #4</td>
<td>0.43</td>
</tr>
<tr>
<td>Shale #5</td>
<td>0.55</td>
</tr>
<tr>
<td>Shale #6</td>
<td>0.29</td>
</tr>
<tr>
<td>Shale #7</td>
<td>0.40</td>
</tr>
</tbody>
</table>
| Note that the index $x_j(j = 1, 2, 3, \ldots, 6)$ represents the normalized value of brittleness, brittle mineral content, cohesion, clay mineral content, angle of internal friction and unconfined compressive strength of shale cores, respectively.
the angle of internal friction and unconfined compressive strength are listed as the equally weakest influencing factors because they influence the fracability through their effects on the brittleness and toughness of rock.

Based on the scale matrix, applying the equation of asymptotic normalization coefficient (ANC), i.e. Eq. (6), yields the array of weight value \( w_j/(j = 1, 2, 3, \ldots, 6) \) of each influencing factor:

\[
W_j = (W_1, W_2, W_3, W_4, W_5, W_6)
\]
\[
= (0.43, 0.3, 0.13, 0.07, 0.03, 0.03)
\]

where the weight values \( W_1 \) to \( W_6 \) correspond to brittleness, brittle mineral contents, cohesion, clay mineral contents, angle of internal friction and unconfined compressive strength of rock.

Accordingly, the fracability of the reservoir shale at various depths can be generally expressed as

\[
P^{(i)} = \sum_{j=1}^{6} w_j^{(i)} x_j^{(i)}(i = 1, 2, 3, \ldots, n), \tag{9}
\]

where \( P^{(i)} \) represents the fracability of the \( i \)th set of shale, \( w_j^{(i)} \) refers to the weight value assigned to the \( j \)th factor of the \( i \)th set of shale, \( x_j^{(i)} \) stands for the normalized value of the \( j \)th factor of the \( i \)th set of shale. Considering our cases and substituting the weight value of each factor, the coefficient of fracability of shale reservoir can be calculated by

\[
P^{(i)} = 0.43x_1^{(i)} + 0.3x_2^{(i)} + 0.13x_3^{(i)} + 0.07x_4^{(i)} + 0.03x_5^{(i)}
\]
\[
+ 0.03x_6^{(i)}(i = 1, 2, 3, \ldots, n). \tag{10}
\]

Substituting the value \( x_j^{(i)} \) of each influencing factor (see Table 6), the coefficients of fracability of shale reservoirs at various depths are obtained, as shown in Fig. 2.

From the expression of shale fracability one can conclude that the coefficient of fracability varies between 0 and 1.0 where the upper limit 1.0 means that shale possesses the best fracability and the lower limit 0 means that shale has the worst fracability. Thus, the result of Fig. 2 indicates that the fracability of the shale investigated in this study decreases with the increase of reservoir depths in general. Although the shales of categories #6 and #7 that were extracted from the formations with the same depth but in different areas have inconsistent physical and mechanical characteristics, their coefficients of fracability are very close. This implies that it may not be appropriate to evaluate the shale fracability using a single argument. The comprehensive quantification method taking into account the contribution of multiple influencing factors to shale fracability is necessary.

5. Experimental validation of the quantification method

5.1. Triaxial hydraulic fracturing tests of shale cores

To validate the efficiency and accuracy of the proposed quantification method for shale fracability, a triaxial hydraulic fracturing test was carried out on the original shale cores to measure the fracture behavior. The cylindrical shale specimens with 108–110 mm in diameter and 100 mm in length belonging to the categories Shale #1 (3317.00–3317.50 m) and Shale #2 (3364.20–3364.35) were selected for the tests. The two categories of shale cores are characterized by the highest level and the lowest level of fracability according to the above evaluation. Fig. 3 shows the hydrofracturing testing device and the shale specimen. The shale cylinder was enclosed by a cement cube with a side length of 290 mm and a height of 350 mm to accommodate the space of the hydrofracturing kit (see Fig. 3b). The horizontal stresses with the maximum value 13 MPa and the minimum value 10 MPa were applied to the specimen to reflect the in-situ geostress conditions of the gas reservoirs. The vertical stress was set to be 20 MPa. The fracture stresses breaking down the Shale #1 and Shale #2 specimens were found to be 18.36 MPa and 36.9 MPa respectively. After hydrofracturing, the fractured shale specimens were taken out of the device and placed on the microfocus CT with a spacial resolution up to 4 \( \mu \)m to perform tomography of the fracture population and distribution in three dimensions. Fig. 4 provides the three-dimensional (3-D) reconstructed CT images of the fractured shale cores revealing the inside fracture patterns. More details about the hydraulic fracturing tests of the shale cores can refer to the literature [45].

5.2. Characterization of fracture network

There is a strong positive correlation between fracture degree and fracability of rock. The fracture characteristics, including population, distribution, and complexity, are generally used to quantify the fracture degree of rock. To quantify the fracture degree of the tested shale cores, we adopted fractal geometry to measure the fractal dimensions of the fractures extracted from the two-
The fractal dimensions reflect the characteristics of fracture population and distribution [49–52]. The more developed and complex the fractures are, the larger the fractal dimension of the fracture network is. A larger fractal dimension of the fracture network indicates a better fracability of shale rock. Fig. 5 shows the binarized 2-D CT image of the fractures extracted from a selected 2-D cross section of the shale specimen using image processing and segmentation methods. Additionally, Fig. 5 presents the box-covering method for calculating the fractal dimension of the fracture network of the shale cores. The fractal dimension of the 2-D fracture network can be obtained using the following equations

\[
D_F = \lim_{{k \to \infty}} \frac{\ln N_{d_k}}{\ln \delta_k},
\]

where \(D_F\) refers to the fractal dimension of the fracture network, \(\delta_k\) is the side length of the \(k\)th covering grid, \(N_{d_k}\) stands for the number of the grids covering the fractures, and \(k\) means the \(k\)th covering. The binarized image of fractures is covered by the grids with a side length \(\delta_k\), and count the number of grids \(N_{d_k}\). Then halve the side of the grid to subdivide the grids \(\delta_k\) into a smaller one \(\delta_{k+1}\), and count \(N_{d_{k+1}}\). Repeating the subdividing steps yields an array of \(N_{d_k}\) corresponding to \(\delta_k\). Plotting the number \(N_{d_k}\) against \(\delta_k\) in a double logarithmic coordinates generates the curve of \(\ln \delta_k - \ln N_{d_k}\), and the negative slope of the curve represents the fractal dimension \(D_F\). A total of 276 2-D CT images of the transverse sections of the specimen were calculated and used for representation of its 3-D network. A computer program was developed to determine the fractal dimensions of the 3-D fracture network based on the 2-D measurement [50,51].

Applying the above methods to the hydrofracturing test results, the fractal dimensions of the fractures of Shale #1 and Shale #2 cores are found to be 2.88 and 2.01, respectively. It means that the Shale #1 core is easier to be fractured than the Shale #2 core. In other words, the Shale #1 core is characterized by a higher fracability than the Shale #2 core. This is in good agreement with the results of the analytical quantification model of fracability, and verifies that the proposed method is effective for quantitatively evaluating the fracability of the shale reservoirs.

6. Conclusions

This study reports an analytical method for quantitative evaluation of the fracability of reservoir shale based on the analysis of contribution of the six influencing factors to fracability and analytic hierarchy process (AHP). A series of natural shale cores were collected from the shale gas formations with varies buried depths in the field for establishing the evaluation model of fracability and verifying efficiency and accuracy of the proposed method. The main conclusions are as follows:

1. The contribution of the six physical and mechanical characteristics of shale rock, i.e. brittleness, mineral content,
cohesion, angle of internal friction, and unconfined compressive strength, to the fracability of reservoir shale were analyzed. The significance of these factors as for the fracability was ranked based on the AHP principles.

(2) To establish the quantitative evaluation model of shale fracability, the natural shale cores were collected from the shale gas reservoirs with various buried depths in the field. The uniaxial and triaxial tests were conducted to determine the properties of the influencing factors of the shale.

(3) Based on the significance analysis and the experimental results of the influencing factors, an analytical model for quantitatively evaluating shale fracability was established using AHP and applied in quantitative evaluation on the reservoir shales with six different depths. The results show that the fracability of the shale investigated in this study decreases with the increase of reservoir depths in general. Despite the physical and mechanical characteristics of the shales extracted from the formations with same depth but in different areas are inconsistent, their coefficients of fracability are very close. This implies that the comprehensive quantification taking into account the contribution of multiple influencing factors to fracability is necessary.

(4) To verify the quantification method, the hydraulic fracturing test was performed on a couple of shale cores to measure and compare their fracture behavior. The fractal geometry was applied for characterizing the fracture degree and complexity. It is shown that a larger fractal dimension of the fracture network corresponds to a better fracability of shale. The more developed and complex the fractures are, the larger the fracability of the shale is. The experimental results coincide with the evaluation of the proposed fracability model.

It is noteworthy that the quantification method for shale fracability is established based on the analysis of significance of the limited influencing factors and the experimental results of the shale cores extracted from specific reservoir formations. Further fracability evaluation and experimental tests on shales from various geographical locations and reservoirs are needed for verifying the accuracy and applicability of this fracability evaluation method.

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