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# NUMERICAL SIMULATION OF THE BRAZILIAN TEST AND THE TENSILE STRENGTH OF ANISOTROPIC ROCKS AND ROCKS WITH PRE-EXISTING CRACKS

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**Abstract:** In the present study, an FEM/DEM coupled approach is used to simulate the fracture process in Brazilian tests. The study was performed using proprietary software to simulate both the intact rock behaviour and the development of fractures. Through the introduction of distinct elements with assigned contact properties, if the fracture criterion within the intact rock (represented by FEM) is met, then a crack (represented by DEM) is initiated. Adaptive re-meshing allows the fracture process through the FEM mesh to be tracked and visualized. In this study, Brazilian tests of rock samples with isotropic and anisotropic properties are simulated. Samples with pre-existing cracks are also investigated to find out the effect of pre-existing cracks on the effective tensile strength of the rock samples. It is found that rock anisotropy and pre-existing cracks (length and orientation) play an important role in determining the behaviour of tensile crack initiation and propagation, and hence the overall tensile strength of the rocks.

**Keywords:** Brazilian test, fracture mechanics, tensile strength, numerical simulation, coupling, ELFEN.

## 1. INTRODUCTION

The Brazilian test was developed to measure the tensile strength of brittle materials like rocks and concrete (Berenbaum and Brodie, 1959). The Brazilian testing procedure is simple and the specimen preparation is easy compared to other test methods. Standard test method had been suggested (ISRM, 1978). The indirect tensile strength of a disc sample ( *Figure 1*) of radius $R$ and thickness $t$, with known load at failure $P$ is given by

$$\sigma_t = \frac{P}{\pi R t} \qquad (1)$$

The stress field inside the disc can be obtained by solving a differential equation that employs Airy's stress function and satisfies the boundary condition of the sample. For example, the elastic solution for an isotropic medium was given in Jaeger & Cook (1979), the three-dimensional correction to the two-dimensional theoretical solution was given in (Wijk, 1978), the solution for transversely isotropic rocks was presented by Chen et al. (1998a). The solution by Chen et al. (1998a) is implicit and numerical techniques are needed to calculate the stress field but the authors provided a set of charts to assist the users to determine the stress concentration factors at the disc centre. Recently, explicit representations of stresses and strains at any point of the anisotropic circular disc compressed diametrically was presented by Exadaktylos and Kaklis (2001), Claesson and Bohloli (2002). Theoretical treatment of tangential loading effects on the Brazilian test stress distribution was given in Lavrov and Vervoort (2002).

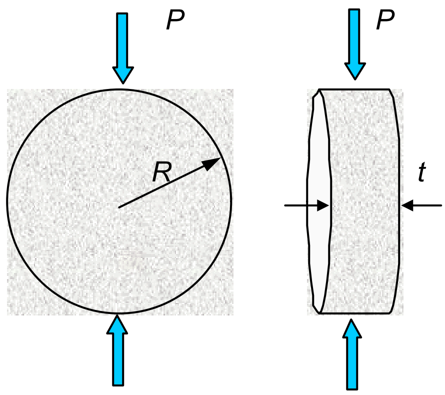



*Figure 1 Brazilian test for indirect tensile strength.*

Layered rocks and rocks with pre-existing cracks are inhomogeneous materials. Theoretical solutions to rock mechanics problems find limited application in engineering practice in dealing with heterogeneous rocks. Hence, many researchers have thus turned to the experimental and numerical approaches to study Brazilian tests on rocks.

After reviewing some of the previous work on this subject, we proposed to use a new numerical tool to simulate the fracture process in Brazilian tests. Three types of samples were selected for testing: rock samples with either isotropic or anisotropic properties and samples with pre-existing cracks. The Brazilian tests were simulated using a FEM/DEM coupled tool.

## 2. REVIEW OF SOME PREVIOUS STUDY ON BRAZILIAN TESTS

Summarized below are some test and numerical simulation studies that employed the Brazilian test scheme to study various special rock mechanics problems.

SEM observation on Brazilian test samples shows that cleavage planes and grain boundaries form preferential fracture paths for the fractures (Van de Steen et al., 2002). They observed that the global direction of the primary fracture in a Brazilian test is perpendicular to the global minor (tensile) principal stress direction. However, locally, the orientation of the grains, the internal structure of the mineral grains and the presence of defects often affect the direction of the fracture.

Brazilian tests on anisotropic sandstones (Chen et al., 1998a) and Dionysos marbles (Exadaktylos and Kaklis, 2001) were tested to illustrate that the analytical solution can be effectively used as a "back-analysis" tool for the characterization of rock elasticity and tensile strength properties. The Brazilian test, which is normally used for the determination of rock tensile strength, can be applied to measure rock fracture toughness (Guo et al., 1993). Samples with a centre notch were utilized to examine the fracture mode in Brazilian tests by Jia et al. (1996), by changing the notch inclination angle, with respect to the loading direction, the mode of fracture was varied from mode I (tensile) to mixed mode (tension-shear and compression-shear).

Malan et al. (1994) tested quartzite samples with an interface running perpendicular or nearly perpendicular to the diametral axis. The slip of the interface and the local asperities affect the fracture patterns. Lavrov et al. (2002) carried out experimental study to investigate the Kaiser effect in cyclic loading tests of disc specimens of a brittle limestone in diametrical compression with acoustic emission measurement. It was found that rotation by more than $10^o$ resulted in complete disappearance of the effect. This is because the previous loading generated fractures have preferred orientations. If the loading direction is changed ($>10^o$), then, tensile cracks will be generated in the direction parallel to the current loading direction.

Numerous studies of Brazilian tests have concentrated on the numerical simulation of crack initiation and propagation process. The Boundary Element Method (BEM) has been used to simulate the cracking process in the Brazilian test by some researches (Malan et al., 1994; Chen et al., 1998b; Wang and Xing, 1999; Van de Steen et al., 2002; Lavrov et al., 2002). "Seed points" are required in the medium arbitrarily or at the junction between existing elements or at the tips of growing fracture segments (Malan et al., 1994). In the works by Lavrov et al. (2002) and Van de Steen et al. (2002), cracks are introduced only along the elements of a tessellation mesh, which is generated before the simulation as a Voronoi cell structure with an internal triangulation. The elements of the mesh thus indicate potential sites for the formation of cracks in the medium. The crack path thus depends on the mesh shape and density.

The Finite Element Method (FEM) has been employed to study the fracture propagation in the Brazilian test. Re-mesh techniques are used to update the mesh following the crack extension (Bouchard et al., 2000). This technique becomes less efficient when multiple cracks propagate at the same time. One distinct advantage of FEM over BEM is that it can model the material heterogeneity more easily. Material heterogeneities can have significant effects on the tensile stress distribution along the loading axis of the Brazilian indirect tensile tests. The material heterogeneity is considered using Weibull distribution randomly assigned to cell meshes (Tang et al., 2001). The use of cell mesh (very fine mesh) also eliminates the need for the continuous update of re-meshing; the cracking elements are assigned very low stiffness and strength parameters, representing open fractures. To produce realistic fracture images observed from the experiments, an extremely fine mesh has to be used, which is computationally ineffective. The random distribution of material properties may affect the fracture path. To account heterogeneity accurately, Yeu et al. (2003) used digital image processing techniques to identify and classify the main homogeneous material types and their distribution structures and demonstrated the method in the FEM analysis of a Brazilian indirect tensile test in rock mechanics and pavement engineering. One problem associated to the FEM method for fracture propagation simulation is that it lacks the crack contact mechanism. The contact mechanism is vital to crack propagation when the material changes from a continuum state to a discontinuum state.

The Distinct Element Method has been used to simulate the fracture process in a Brazilian test. The proprietary software package $PFC^{2D}$, developed by Itasca Consulting Group Inc. (Itasca, 2002), is widely used (Diederichs, 1999; Wang et al., 2003). Because $PFC^{2D}$ represents the rock as an assembly of circular discs that are bonded to one another, the simulated fracture pattern rarely resembles the real case. Nevertheless, this type of simulation provides a tool to study the micromechanics that governs the tensile strength of rocks.

## 3. NUMERICAL SIMULATION OF BRAZILIAN TEST USING AN FEM/DEM APPROACH

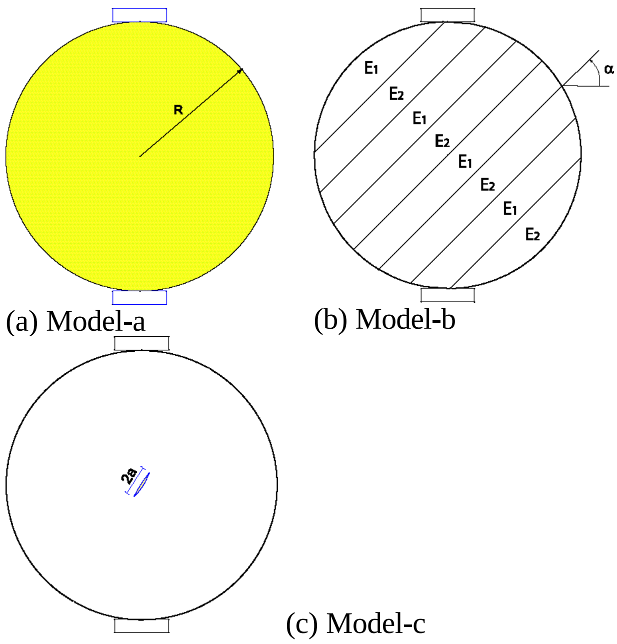
### *3.1 ELFEN*

The proprietary finite/discrete element code ELFEN, developed by Rockfield Software Ltd (2002), was recently acquired by MIRARCO. ELFEN provides a unique ability to model fracture initiation and growth as well as the extent of damage around excavations under static and dynamic loading conditions. ELFEN itself can be used just like Itasca's $PFC^{2D}$ and $PFC^{3D}$ codes, as it too contains disc and ball elements that incorporate all the contact rules that the PFC codes have. The unique feature of ELFEN is that it simulates the transition of a rock mass from a continuum state to a discontinuous state seamlessly. If the fracture criterion within the intact rock (represented by FEM) is met, then a crack (represented by DEM) is initiated. Adaptive re-meshing allows the fracture process through the FEM mesh to be tracked and visualized; thus contact properties can be assigned to pre-existing cracks and newly generated cracks.

### *3.2 Simulation models*

In this study, ELFEN is used to simulate Brazilian tests of rocks. The Rankine rotating fracture model, which has two distinct material parameters, tensile strength and fracture energy, is used in our simulation. This model is designed for modelling the tensile failure of brittle materials such as rock, glass, and ceramic. The initial failure surface is defined by a tension failure surface. Post initial yield, the rotating crack formulation represents the anisotropic damage evolution by degrading the elastic modulus in the direction of the major principal stress invariant. The damage parameter is dependent on the fracture energy. Stresses are computed in the FEM elements and checked against the fracture criterion in each loading step. This explicit dynamic finite/discrete element tool enables the users to track the fracturing process in a unique fashion.

Three rock types are simulated in this study: Model-a represents homogeneous, isotropic rocks (Figure 2a), Model-b represents sedimentary rocks with inter-layered rocks of different material properties (Figure 2b), and Model-c contains pre-existing cracks of different lengths and orientations (Figure 2c). Material properties listed in Table 1 are assigned to Models a to c. For Models a and c, the properties of Rock 1 are assigned. In Model-b, inter-bedded layers are assigned the Rock 1 and Rock 2 material properties alternatively. The heterogeneity in Model-b is represented by elastic modulus difference only. The strengths of the inter-bedded layers are assumed the same. The elastic modulus of the platen is assigned to a very high value so that the stiff loading condition is valid.




(a) Model-a (b) Model-b

(c) Model-c

*Figure 2. Simulation models: (a) Homogeneous rocks; (b) Layered rocks; (c) Rocks with pre-existing cracks. R = 50 mm and t = 1 mm for all models.*

*Table 1. Rock material properties.*

| | E (GPa) | n | $s_t$ (MPa) | $G_f$ |
|---|---|---|---|---|
| Rock 1 | 50 | 0.25 | 5 | 0.05 |
| Rock 2 | 25 | 0.25 | 5 | 0.05 |

Note: E –Young's modulus; ν –Poisson's ratio; $s_t$ –tensile strength; $G_f$ –fracture energy (N/mm)

## 3.3 Results

### 3.3.1 Homogeneous rocks

The platen is applied a constant vertical velocity to simulate applied loading. The reaction force at the platen and the vertical displacement are recorded (Figure 3). Figure 3 shows that it takes about 0.02 mm for the platen to move before it contacts with the rock sample and the force starts to build up. The non-smoothness of the curve represents the influence of loading rate on the solution. The explicit solution algorithm is adopted in the fracture simulation in ELFEN so that smaller loading rate tends to give smoother curves but at the cost of longer computation time.

Linear force-displacement relation is seen before peak load in Figure 3, which agrees to the experimental results. Cracks parallel to the loading direction starts to appear when the peak load is reached. The crack propagation process can be tracked in details by the numerical tool. Cracks propagate to both loading platens right after the peak load as shown as inserts in Figure 3. The effective tensile strength calculated from Eq. (1) is 5.45 MPa, which is slightly higher than the assigned tensile strength of 5 MPa. This is because another parameter, fracture energy, is used in ELFEN for fracture simulation. High fracture energy contributes to higher global peak load.

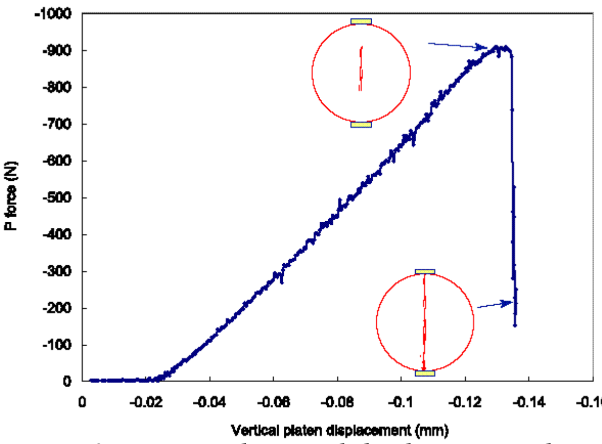



*Figure 3. Force and vertical displacement relation of isotropic rocks in a Brazilian test simulation.*

### 3.3.2 Sedimentary rocks (layered rocks)

Sedimentary rocks such as sandstones, limestones and shales may have beds or strata sedimentary structures. As explained before, the numerical simulation of Brazilian tests is performed for this rock type by assigning different moduli to rock layers. The effect of the bedding orientation is also investigated. The bedding boundaries are assumed to be perfectly bonded.

It is observed that the tensile crack appears in the disc centre when the peak load is reached. However, as compared to the homogeneous rock samples, the crack propagation paths are affected by the bedding orientation (Figure 4). The paths are not straight but zigzagged except when $\alpha = 0^o$ and $\alpha = 90^o$. The fracture pattern for $\alpha = 75^o$ shows the largest deviation from the others. At $\alpha = 75^o$, cracks are developed in Rock 2 material in the centre of the disc in the bedding direction and gradually converge to the platens.

The effective tensile strengths for various bedding orientations and their strength anisotropy are evident from the simulation results (Figure 5). The tensile strength first decreases from $\alpha = 0^o$ to $\alpha = 45^o$ and then gradually increases to a peak value at $\alpha = 60^o$. The strength decreases again as the bedding inclination angle increases from $\alpha = 60^o$ to $90^o$, where the minimum tensile strength is observed at $\alpha = 90^o$. At $\alpha = 90^o$, the loading platen crosses the two rock layers with different moduli. Stress in the high modulus layer is higher thus the tensile failure initiates at an earlier stage.

The loading contact condition is found to affect the fracture pattern in this case. In our simulation, it is found that if the lateral dimension of the loading platen is not fixed, the failure will initiate directly under the loading points in some cases. Similar experimental observation was made by Hudson et al. (1972).

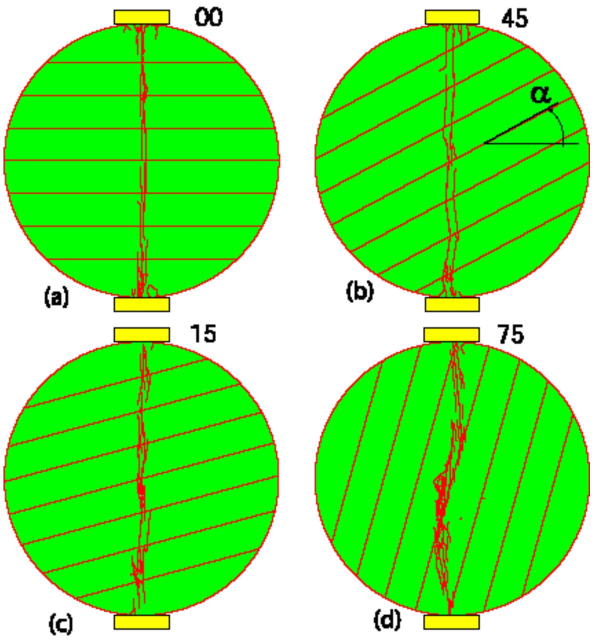



*Figure 4. Fracture propagation in a disc with layered rocks inclined at different angles.*

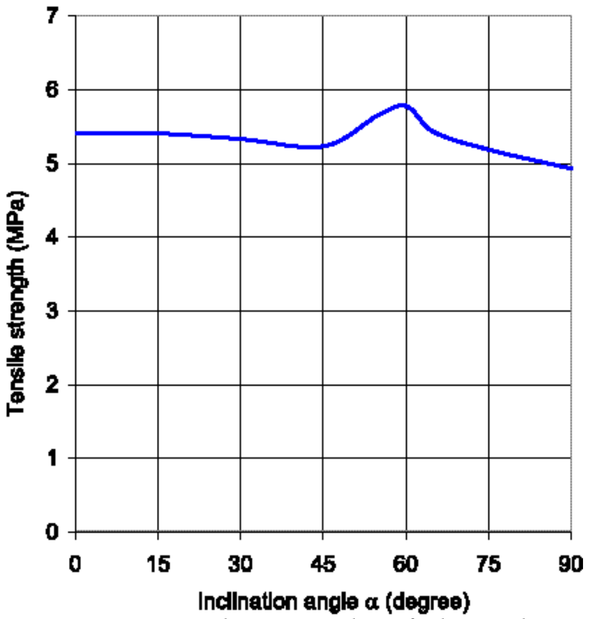



*Figure 5. Tensile strength of layered rocks determined from the Brazilian tests.*

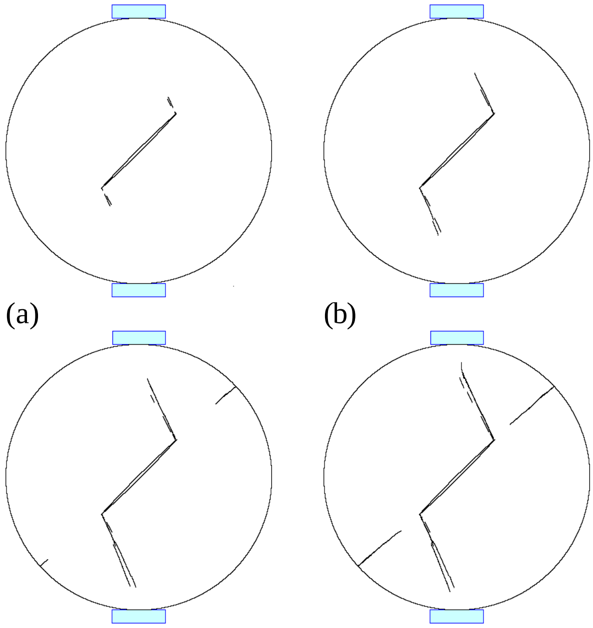



*Figure 6. Fracture initiation and propagation in a disc with a 40 mm long pre-existing inclined crack at the disc centre, oriented at 45°.*

### 3.3.3 Samples with pre-existing cracks

Brazilian tests simulation of rock samples with pre-existing cracks is executed with the crack length and orientation taken as variables. One example of crack initiation and propagation is presented in Figure 6 for a 40 mm long crack oriented at 45°. New cracks are not generated in the disc centre anymore. They emerge at the pre-existing crack tips in a direction of about 72°, measured from the direction of the pre-existing crack. Visible new cracks are generated right after the global peak in the load – displacement curve. The wing cracks gradually propagate and at the load level of about 50 % of the peak, another pair of cracks that are inline with the pre-existing cracks starts to appear from the disc boundary (Figure 6c). As loading continues, the wing cracks propagate towards the loading platen and the inline cracks propagate towards the disc centre (Figure 6d). Note that the inline cracks are only generated when the pre-existing crack length is greater than a threshold. In our simulation, the inline cracks are observed for a crack length greater than 40 mm in a disc whose diameter is 100 mm.

The effective tensile strengths calculated from Eq. (1) for samples with different length of pre-existing cracks oriented at 45° are presented in Figure 7. It is seen that the macro tensile strength decreases as the pre-existing crack length increases.

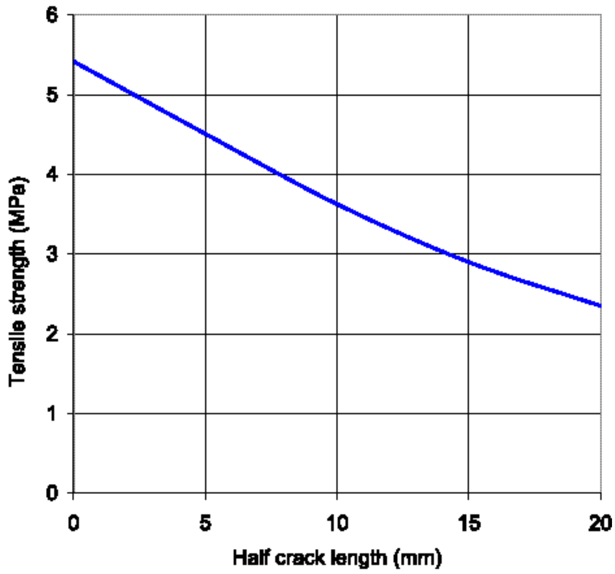



*Figure 7. Tensile strength of rocks with pre-existing cracks of difference lengths at the disc centre, oriented at 45°.*

## 4. CONCLUSIONS

The numerical simulation tool used for the present study captures the dominant characteristics of the rock failure process in a Brazilian test. The fracture pattern and the tensile strength of rocks are affected by rock material property, anisotropy, and pre-existing cracks. The approach adopted here provides a unique way to study various special rock mechanics problems.

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