计算固体力学

(Computational Solid Mechanics)

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课程内容

1. 计算力学和有限元法
2. 有限元法的理论基础
3. 弹性力学问题的有限元求解格式

3 Finite element solution process

- The weak form of the equilibrium equations for linear elasticity may be written using variational notation as

$$G(\delta\mathbf{u},\mathbf{u},\boldsymbol{\sigma}) = \int_\Omega \delta\mathbf{u}^{\mathrm{T}}\left(\rho\ddot{\mathbf{u}} - \mathbf{b} - \boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\sigma}\right)\mathrm{d}\Omega = 0$$

Equilibrium equations $\boldsymbol{\mathcal{S}}^{\mathrm{T}}\boldsymbol{\sigma} + \mathbf{b} = \rho\,\ddot{\mathbf{u}}$

- Expanding the equations for the three-dimensional problem in Cartesian coordinates gives

$$G(\delta\mathbf{u},\mathbf{u},\boldsymbol{\sigma}) = \int_\Omega \begin{Bmatrix}\delta u\\ \delta v\\ \delta w\end{Bmatrix}^{\mathrm{T}}\left(\rho\begin{Bmatrix}\ddot{u}\\ \ddot{v}\\ \ddot{w}\end{Bmatrix} - \begin{Bmatrix}b_x\\ b_y\\ b_z\end{Bmatrix} - \begin{Bmatrix}\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}\\ \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}\\ \frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_z}{\partial z}\end{Bmatrix}\right)\mathrm{d}\Omega = 0$$

3 Finite element solution process

1. Define the problem to be solved in terms of differential equations. Construct the integral form for the problem as a virtual work, variational, or weak formulation.
2. Select the type and order of finite elements to be used in the analysis.
3. Define the mesh for the problem. This involves the description of the node and element layout, as well as the specification of boundary conditions and parameters for the formulation used.

4

3 Finite element solution process

4. Compute and assemble the element arrays. The particular virtual work, variational, or weak form provides the basis for computing the specific relationships of each element.
5. Solve the resulting set of linear algebraic equations for the unknown parameters. For static problems this requires the solution to $\mathbf{K}\,\tilde{\mathbf{u}} = \mathbf{f}$.
6. Output the results for the nodal and element variables. Graphical outputs also are useful for this step.

5

The End