1 Introduction of Finite Element Method

1.1 Development process of finite element method

1.2 Computation procedure of finite element method

1.3 Main contents of the book

1.4 Exercises

1.2 Computation procedure of finite element method

Computational mechanics is the discipline concerned with the use of computational methods to study phenomena governed by the principles of mechanics. Before the emergence of computational science (also called scientific computing) as a "**third way" besides theoretical and experimental sciences**, computational mechanics was widely considered to be a sub-discipline of applied mechanics. It is now considered to be a sub-discipline within computational science.

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计算力学 锁定

本词条由"科普中国"百科科学词条编写与应用工作项目 审核 。

科普中国 CHINA SCIENCE COMMUNICATION

Bai百科

计算力学（computational mechanics）是根据力学中的理论，利用现代电子计算机和各种数值方法，解决力学中的实际问题的一门新兴学科。它横贯力学的各个分支，不断扩大各个领域中力学的研究和应用范围，同时也在逐渐发展自己的理论和方法。计算力学的应用范围已扩大到固体力学、岩土力学、水力学、流体力学、生物力学等领域。计算力学主要进行数值方法的研究，如对有限差分方法、有限元法作进一步深入研究，对一些新的方法及基础理论问题进行探索等等。计算力学横贯各个力学分支，为它们服务，促进它们的发展，同时也受它们的影响。

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1.2 Computation procedure of finite element method

- **Numerical methods in computational mechanics**

作为力学分支的计算力学，发展了有限元(finite element method, FEM)、离散元(discrete element method, DEM) 、有限差分法(finite difference method, FDM)、无网格法(mesh-less method, MLM)、扩展有限元法(extended finite element method, XFEM)、边界元(boundary element method, BEM)、半解析方法(semi-analytic methods）等理论和方法，为虚拟仿真提供了工具。

有限元分析是虚拟设计的基本组成部分，它提供了更快捷和低成本的方式评估设计的概念和细节，使得人们越来越多地应用数值试验代替样品原型的物理试验。

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1.1 Development process of finite element method

- **The treatment of "standard discrete problems".**

1. The continuum is divided into a finite number of elements whose behaviour is specified by a finite number of parameters.

离散

2. As an assembly of its elements, the solution for the complete system precisely follows the same rules as those applied to standard discrete problems.

集成

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1.2 Computation procedure of finite element method

1. **Define the problem to be solved by the governing differential equations.** Based on the differential equations in the analysis of continuous systems, the equivalent integration form of the problem is constructed as virtual work, variational or weak form.
2. **Select the type and order of the finite element.** These elements and corresponding shape functions will be used in the analysis of discrete systems, which will be substituted into the equivalent integral form.
3. **Define a set of mesh for the problem.** This involves the description of the distribution of nodes and elements and the description of the boundary conditions.

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1.2 Computation procedure of finite element method

4. **Compute and assemble the matrixes from elements.** The relationships in all elements are defined and assembled by the equivalent integral form of the problem; for example, the global stiffness matrix and load vector are set from each element using the element location vector. In this way, the continuous system is meshed and discretized to form an approximate discrete system. In this way, the differential equations representing continuous systems are transformed into algebraic equations of discrete systems.

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1.2 Computation procedure of finite element method

5. **Solve the system of algebraic equations.** For widespread static problems, the algebraic equations are especially linear, which could be solved effectively by some well-developed mathematical techniques or specialized solvers. The solutions of the system of algebraic equations are solutions on all nodes.
6. **Derive the solutions of variables in the global domain.** Using the solutions on the nodes and the solutions of displacement, stress, and strain (derivatives of displacement) in each element domain can be obtained by further applying the interpolation of shape functions.

1.2 Computation procedure of finite element method

Continuous system	Equivalent transformation	Continuous system	Discretization	Discrete system
Differential equations		**Integral equation**		**Algebraic equations**
$Lu=0$	Weak form	$G(w, u)=0$	Element and shape function	$Ku=f$
L: Differential operator		G: Integral operator		K: Stiffness matrix
u: Unknown displacement		w: Arbitrary function		f: Load vector
		u: Unknown displacement		u: Unknown displacement

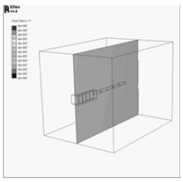
- **Computation procedure from differential equations to integral equation and then to algebraic equations**

1.2 Computation procedure of finite element method

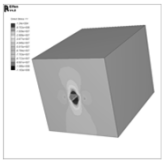
■ **Mining engineering——coal mining**

- **Tunnel excavation**

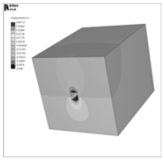
- **3D numerical model**



- **Evolution of Y-stress**



- **Y-stress field**



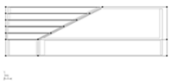
- **Y-displacement field**

1.2 Computation procedure of finite e

■ **Geotechnical engineering——slope**

- **广东梅大高速塌方, 2024.5.1**

- **Slope excavation**



- **2D numerical model**

- **2D structural mesh**

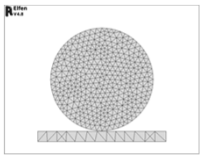
- **Y-stress field**

- **Y-displacement field**

1.2 Computation procedure of finite element method

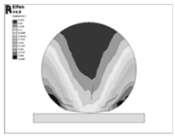
■ **Rock mechanics——crack initiation and propagation**

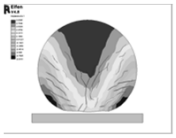
- **Rock disc model**



- **2D numerical model**

- **Crack propagation**



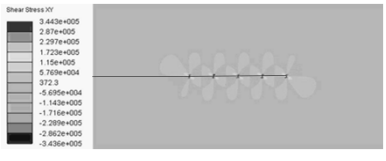


- **Displacement field**

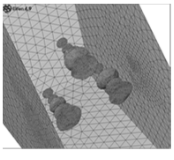
1.2 Computation procedure of finite element method

■ **Petroleum Engineering——hydraulic fracturing**

- **Hydraulic fracturing of tight shale reservoir**



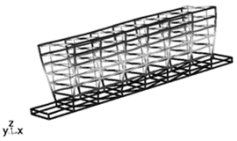
- **Numerical simulation**

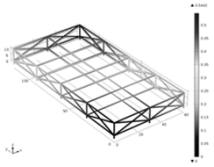


- **Multiple wells fracturing**

1.2 Computation procedure of finite element method

■ Civil engineering——deformation of buildings





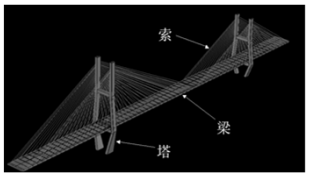
- Deformation of high-rise building
- Deformation of single-story industrial building

1.2 Computation procedure of finite element method

■ Civil engineering——vibration of cable-stayed bridge

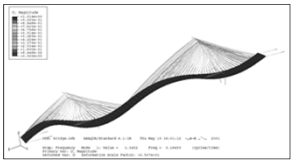
- 施工过程仿真和分析
- 结构动力响应仿真和分析
- 地震响应时程仿真和分析

荆州长江大桥：全长4397.6m ，接线长4444m ，桥面宽24.5米，其中斜拉桥部分宽度26.5米，双向四车道。大桥由北岸的引桥、荆江大堤桥、北岸滩桥、北汊通航孔桥、三八洲桥、南汊通航孔桥、南岸滩桥、荆南干堤桥和南岸引桥等九个部分主组成，总投资13.73亿元。北汊桥主桥为双塔双索面混凝土梁斜拉桥，是目前中国主大跨径的混凝土梁斜拉桥，居同类桥梁世界第二。

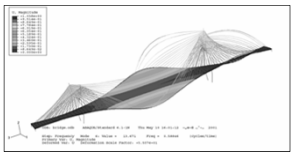


1.2 Computation procedure of finite element method

■ Civil engineering——vibration of cable-stayed bridge



- 1st-oder mode



- 6th-oder mode

- Frequencies

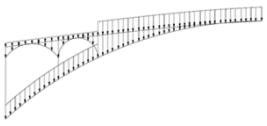
1 0.18459(纵漂)
2 0.28296(一阶对称竖弯)
3 0.36452(一阶对称侧弯)
4 0.47058(一阶反对称竖弯)
5 0.51384(塔反对称侧弯)
6 0.58846(对称扭转)
13 0.66620(竖弯)
32 0.77994(塔对称侧弯)
46 0.82465(竖弯)
50 0.84960(反对称扭转)

1.2 Computation procedure of finite element method

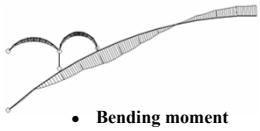
■ Structure optimization——Zhaozhou Bridge

- Zhaozhou Bridge(赵州桥)

该桥是一座空腹式的圆弧形石拱桥，是中国现存最早、保存最好的巨大石拱桥。赵州桥是入选世界纪录协会世界最早的敞肩石拱桥，创造了世界之最。桥长50.82米，跨径37.02米，券高7.23米，两端宽9.6米，桥的设计完全合乎科学原理，施工技术更是巧妙绝伦。唐朝的张嘉贞说它“制造奇特，人不知其所以为”。



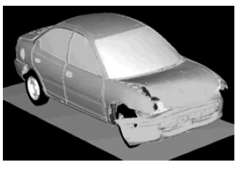
- Model and load



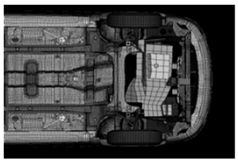
- Bending moment

1.2 Computation procedure of finite element method

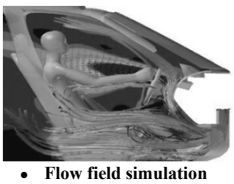
■ Mechanical Engineering - vehicle crash simulation



- Numerical model



- Mesh



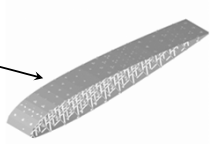
- Flow field simulation

在汽车设计领域中，对初期设计概念和最终设计细节的评估，碰撞的仿真正代替整车的试验如布置判定气囊释放的加速计，内部的缓冲装置，以及选择材料和满足碰撞准则的构件截面。

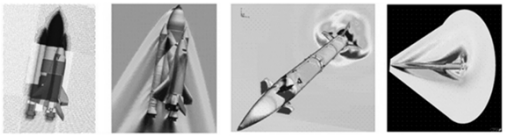
1.2 Computation procedure of finite element method

■ Aerospace——airplane and aerocraft

- Airplane simulation



- 点阵材料翼型结构的拓扑优化

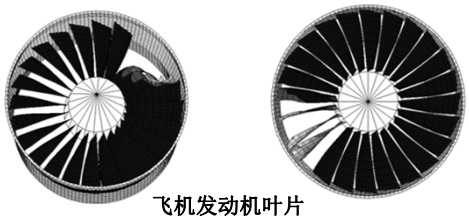


- Aerocraft simulation

1.2 Computation procedure of finite element method

■ Aerospace——bird strike

鸟撞试验:
1814g bird, 200m/s velocity



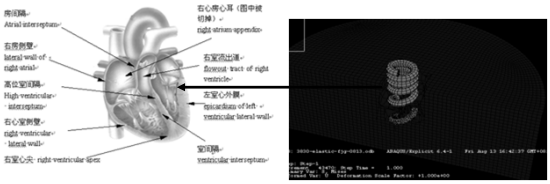
飞机发动机叶片

- 1 msec(兆秒)
- 5 msec(兆秒)

1.2 Computation procedure of finite element method

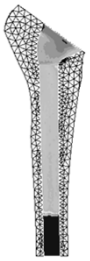
■ Biomechanics——organ

心脏起搏器，心脏肌肉材料

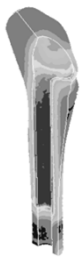


1.2 Computation procedure of finite element method

■ Biomechanics——bone



Distribution of contact pressure in femur.

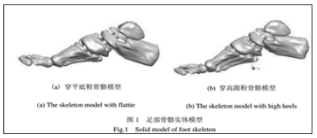


Maximum principal stress in femur due to interference fit.

- Model and mesh

1.2 Computation procedure of finite element method

■ Biomechanics——shoes



(a) 穿平底鞋骨骼模型 (a) The skeleton model with flattie (b) 穿高跟鞋骨骼模型 (b) The skeleton model with high heels

图 1 足部骨骼实体模型 Fig.1 Solid model of foot skeleton

(a) 穿平底鞋模型 (a) The model with flattie (b) 穿高跟鞋模型 (b) The model with flattie

图 2 足部骨骼有限元模型 Fig. 2 The finite element model of foot skeleton

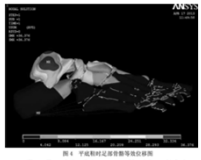


图 4 平底鞋时足部骨骼等效位移图 Fig. 4 The equivalent displacement map of foot skeleton with flattie

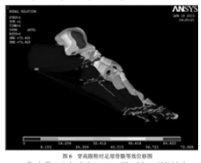


图 5 穿高跟鞋时足部骨骼等效位移图 Fig. 5 The equivalent displacement map of foot skeleton with high heels

穿高跟鞋脚骨应力更高、位移（变形）更大！

表 2 两种情况下的应力、应变结果 Tab. 2 The stress and strain results of this two conditions

状态	SMX	SMX 的位置	DMX	DMX 的位置
穿平底鞋	697.325	距骨下表面	36.376	距骨前表面
穿高跟鞋	1459	第二距骨末端	72.928	舟骨右端

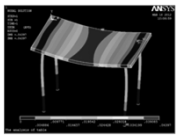
注：SMX 为最大应力值，DMX 为最大等效位移

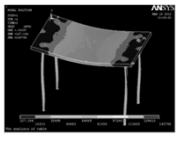
1.2 Computation procedure of finite element method

■ Simulation of desk

假设桌子的四只脚同地面完全固定，桌子上存放物品，物品产生的均匀分布压力作用在桌面，模拟分析桌子的变形和应力。

- Physical model

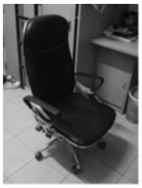


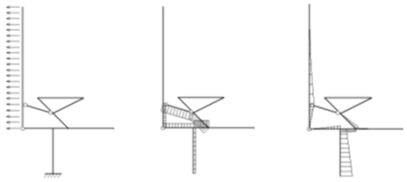


- Displacement
- Von-Mises stress

1.2 Computation procedure of finite element method

■ Simulation of chair

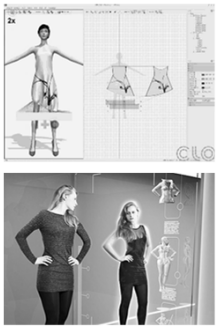


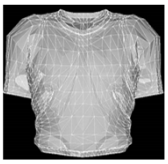


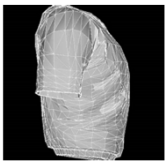
- Physical model
- Numerical model
- Axial force
- Bending moment

1.2 Computation procedure of finite element method

- **Simulation of dressing**







- 3D virtual fitting system
- Von-Mises stress

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1 Introduction of Finite Element Method

1.1 Development process of finite element method

1.2 Computation procedure of finite element method

1.3 Main contents of the book

1.4 Exercises

1.3 Main contents of the book

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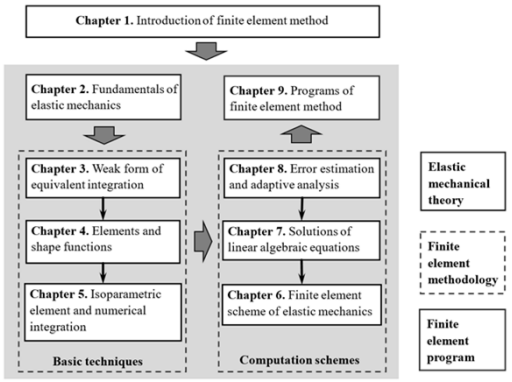

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