

3.3 Finite element computation based on weak form

$$u(x) \approx \hat{u}(x) = \sum_{n=1}^N \phi_n(x) u_n + u_p(x) \quad ? \quad \hat{u}^e = N_1(x') \hat{u}_1^e + N_2(x') \hat{u}_2^e$$

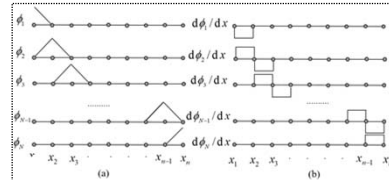


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$$u(x) \approx \hat{u}(x) = \sum_{n=1}^N \phi_n(x) \alpha_n + u_p(x) \quad ? \quad \hat{u}^e = N_1(x') \hat{u}_1^e + N_2(x') \hat{u}_2^e$$

$$\begin{aligned} \hat{u}(x) &= \sum_{n=1}^N \phi_n(x) \alpha_n \\ &= \phi_1(x) \alpha_1 + \phi_2(x) \alpha_2 + \phi_3(x) \alpha_3 + \dots + \phi_{N-1}(x) \alpha_{N-1} + \phi_N(x) \alpha_N \\ &= \phi_1(x) u_1 + \phi_2(x) u_2 + \phi_3(x) u_3 + \dots + \phi_{N-1}(x) u_{N-1} + \phi_N(x) u_N \end{aligned}$$

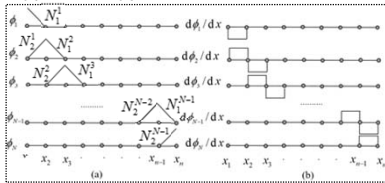


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$$\begin{aligned} \hat{u}(x) &= \phi_1(x) u_1 + \phi_2(x) u_2 + \phi_3(x) u_3 + \dots + \phi_{N-1}(x) u_{N-1} + \phi_N(x) u_N \\ &= (N_1^1) u_1 + (N_2^1 + N_1^2) u_2 + (N_2^2 + N_1^3) u_3 + \dots + (N_2^{N-2} + N_1^{N-1}) u_{N-1} + (N_2^{N-1}) u_N \\ &= (N_1^1 u_1 + N_2^1 u_2) + (N_2^2 u_2 + N_2^2 u_3) + \dots + (N_1^{N-1} u_{N-1} + N_2^{N-1} u_N) \\ &= (N_1^1 u_1^1 + N_2^1 u_2^1) + (N_2^2 u_1^2 + N_2^2 u_2^2) + \dots + (N_1^{N-1} u_1^{N-1} + N_2^{N-1} u_2^{N-1}) \\ &= \hat{u}^1(x) + \hat{u}^2(x) + \dots + \hat{u}^{N-1}(x) \end{aligned}$$



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